

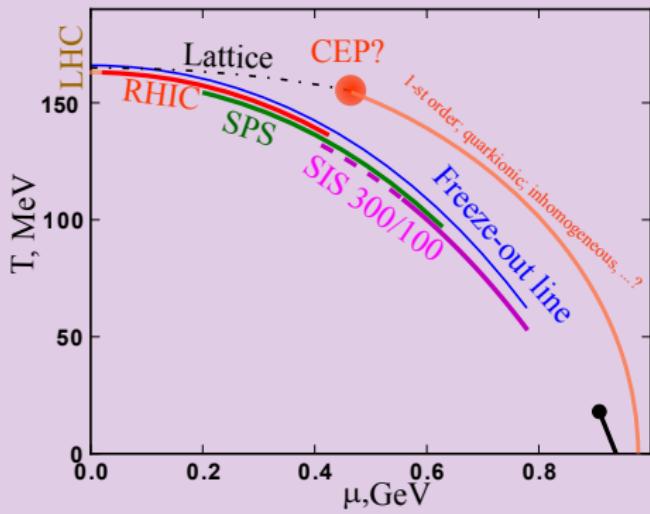
CHARGE FLUCTUATIONS IN EFFECTIVE CHIRAL MODELS

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BNL
from Oct 1

3.10.2011

PHASE DIAGRAM



Schematic phase diagram

(Expected) Structure of the phase diagram

- crossover at small μ_B , underlying O(4) universality class
- critical end point, 3d Ising model universality class
- first-order transition

Fluctuations of conserved charges
~ structure of phase diagram

FLUCTUATIONS OF CONSERVED CHARGES

- **HIC**: conserved charges do not fluctuate. Appropriate cuts of phase space! Talk by Marlene.
- **Exp.:** $\mathcal{P}(N_B) \sim \langle (N_B - \bar{N}_B)^n \rangle \sim$ cumulants
- **Th.:** Grand canonical pressure $p \propto$ generating functional for cumulants

$$\text{E.g.: } \langle (N_B - \bar{N}_B)^4 \rangle - 3\langle (N_B - \bar{N}_B)^2 \rangle^2 = VT^3 \frac{\partial^4 p}{\partial \mu_B^4}$$

- Fluctuations of net-quark number χ_n^q and net-baryon charge χ_n^B

$$\chi_n^q = \frac{\partial^n(p/T^4)}{\partial(\mu_q/T)^n} \quad | \quad \chi_n^B = \frac{\partial^n(p/T^4)}{\partial(\mu_B/T)^n} = \left(\frac{1}{3}\right)^n \chi_n^q$$

- Fluctuations of electric charge χ_n^Q

$$\chi_n^Q = \frac{\partial^n(p/T^4)}{\partial(\mu_Q/T)^n}$$

Hadron Resonance Gas Model ($\mu_S = \mu_Q = 0$):

- $p/T^4 = \sum_i f(m_i/T) \cosh(\mu_B/T) + g(T)$
- $\chi_{2n} \propto \cosh(\mu_B/T) \quad \chi_{2n+1} \propto \sinh(\mu_B/T)$
- $\chi_{2n}/\chi_2 = 1 \quad \chi_{2n+1}/\chi_1 = 1$
- $\chi_{2n} > 0$

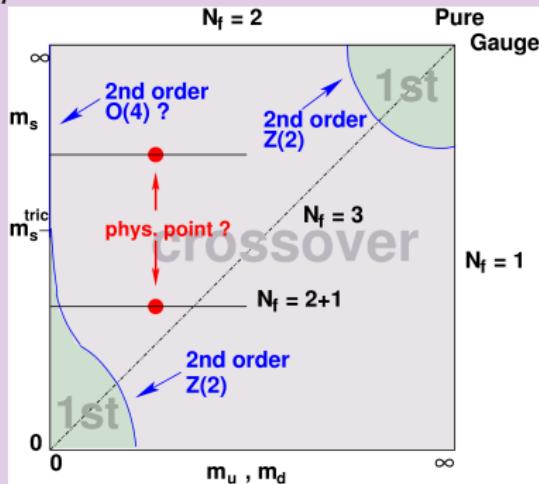
LATTICE QCD RESTRICTIONS

- only zero baryon chemical potential
- lacking continuum limit for susceptibilities

~ modeling QCD

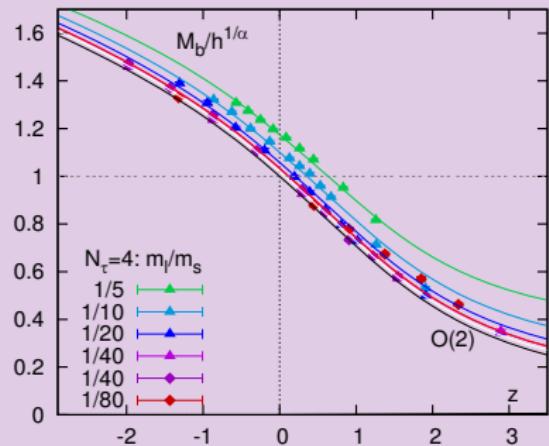
QCD PHASE DIAGRAM: QUARK MASS DEPENDENCE

$\mu = 0$



- QCD at physical m_q ?
- $m_l \rightarrow 0$: O(4) or Z(2)?

F. Karsch et. al. LGT QCD:



- QCD at physical m_π in O(4) scaling
- $N_\tau = 8$ supports these results

Main properties required from models

- **O(4) symmetry in limit of vanishing mass for light quarks**
- simulation of confinement properties (ratios of cumulants are sensitive to degrees of freedom)
- **Beyond mean-field**
 - ↗ non-trivial critical exponents

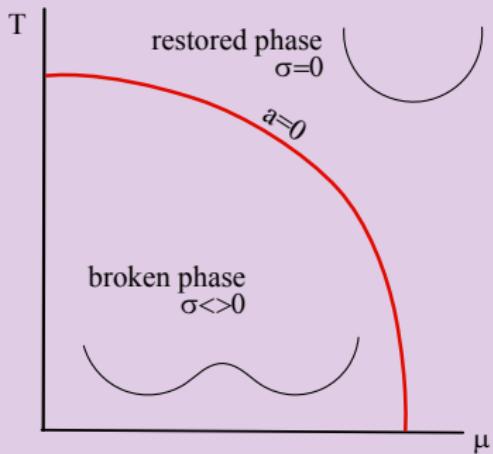
TOY MEAN-FIELD MODEL: NET-QUARK NUMBER FLUCTUATIONS

Landau theory for 2d-order phase transition ($m_\pi = 0$):

$$\Omega = \frac{a}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4$$

σ - order parameter

$$a = \frac{1}{t_0} \left[\left(\frac{T}{T_c} - 1 \right) + \kappa_q \left(\mu_q/T \right)^2 \right]$$



Minimization of Ω : $\partial\Omega/\partial\sigma=0$ leads to
 $\sigma_{\min}^2 = -\frac{a}{\lambda}$ for $a < 0$ and $\sigma_{\min}^2 = 0$ for $a > 0$.

Pressure: $p = -\Omega(\sigma = \sigma_{\min}) = \frac{a^2}{4\lambda}$

Second-order cumulant $\mu = 0$: $\chi_2 \sim (T - T_c)\theta(T - T_c)$

Higher order cumulants $n > 4$ $\chi_n = 0$

BEYOND MEAN-FIELD

Fluctuations of order parameter \sim **non-trivial** exponents

Pressure: $p \sim a^2 \quad \sim p \sim a^{2-\alpha}$

$$a = \frac{1}{t_0} \left[\left(\frac{T}{T_c} - 1 \right) + \kappa_q \left(\mu_q/T \right)^2 \right]$$

α is non-integer number.

3-dimensional O(4) universality class: $\alpha \approx -0.21$

$\mu = 0$: higher cumulants are non-trivial: $\chi_n \sim (T - T_c)^{-\frac{1}{2}(n-4+2\alpha)}$

$\chi_6 \sim 1/(T - T_c)^{1+\alpha} \quad \chi_8 \sim 1/(T - T_c)^{2+\alpha} \quad$ divergent

$\mu \neq 0$: higher cumulants are non-trivial: $\chi_n \sim \left(\frac{\mu}{T}\right)^n a^{-(n-2+\alpha)}$

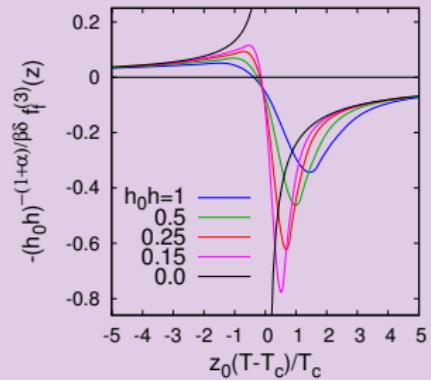
$\chi_4 \sim \left(\frac{\mu}{T}\right)^4 / a^{2+\alpha} \quad \dots$ divergent

BEYOND MEAN-FIELD: O(4) SCALING FUNCTIONS ON LATTICE

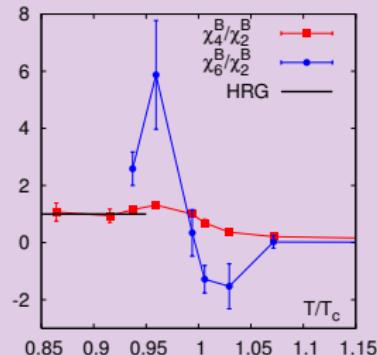
Based on: J. Engels, F. Karsch, arXiv:1105.0584 and
B. Friman et. al., arXiv:1103.3511

Lattice simulations of O(4) models \rightsquigarrow singular part of

$$p/T^4 \propto -f(a, h)/T^4, \quad h \propto m_q$$



$\chi_6(\mu = 0)$



Lattice QCD: C. Schmidt,
2010; Christian's talk

Does singular part dominates in QCD?

POLYAKOV-LOOP EXTENDED QUARK-MESON MODEL

Polyakov loop-extended NJL or QM model

- O(4) symmetry
- quark interaction with Polyakov loops \rightsquigarrow statistical confinement
- in O(4) scaling regime for physical pion mass, as QCD

- Model based on symmetries of QCD

$SU(2)_L \otimes SU(2)_R$, scalar condensate $\sigma \propto \langle \bar{q}q \rangle$

$$Z(3), \text{ Polyakov loop } \ell = \frac{1}{N_c} \langle \text{Tr}_c \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(x, \tau) \right] \rangle$$

- Lagrangian of PQM model:

$$\mathcal{L} = \bar{q} \left[i\gamma^\mu D_\mu - g(\sigma + i\gamma_5 \pi) \right] q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \pi)^2 - U(\sigma, \pi) - \mathcal{U}(\ell, \ell^*)$$

$\mathcal{U}(\ell, \ell^*)$ – Z(3)-invariant Polyakov loop potential

Gluons are coupled to quarks q by covariant derivative

$$D_\mu = \partial_\mu - iA_\mu$$

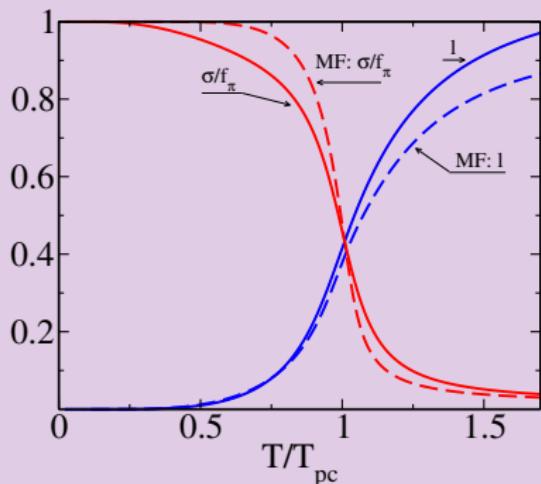
$$U(\sigma, \pi) = \frac{\lambda}{4} (\sigma^2 + \pi^2 - v^2)^2 - c\sigma \text{ is meson potential}$$

FUNCTIONAL RENORMALIZATION GROUP

- accounts for universal critical behaviour near chiral transition
- reproduces scaling properties and critical exponents
- respects symmetries (Goldstone theorem fulfilled, second-order phase transition for $O(N)$ model)

ORDER PARAMETERS

Order parameters in FRG and mean-field approximation:

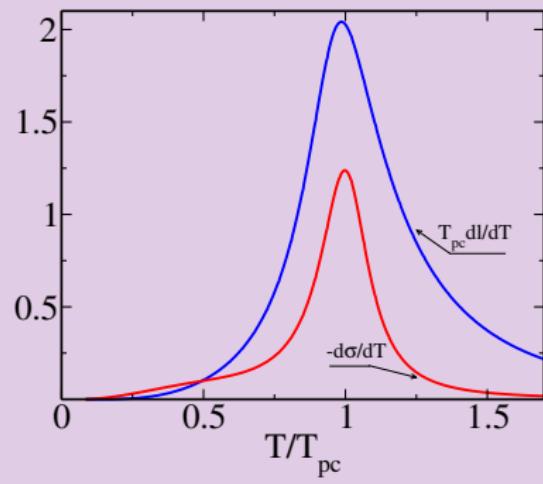


Mean-field: dashed lines

FRG: solid lines

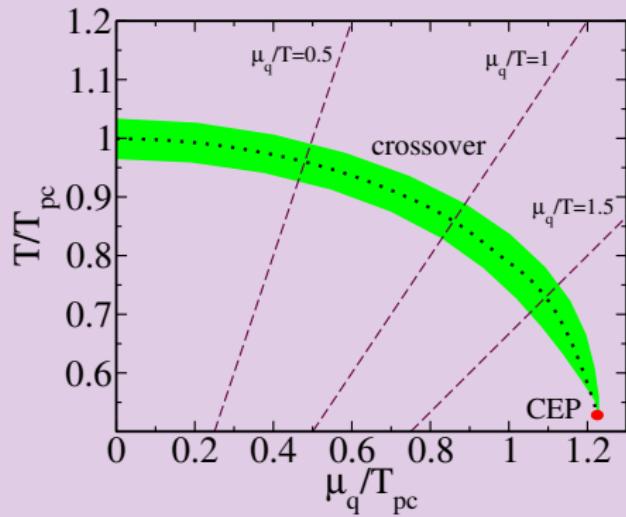
Meson fluctuations: smoothen chiral transition

Temperature derivatives of the order parameters:



T_{pc} is pseudocritical temperature at $\mu_q = 0$

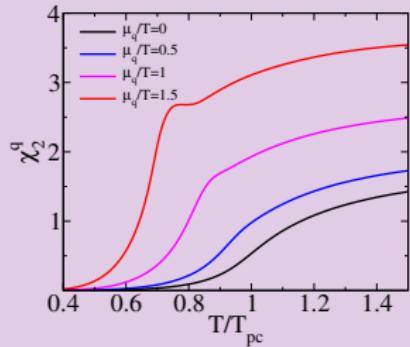
PHASE DIAGRAM



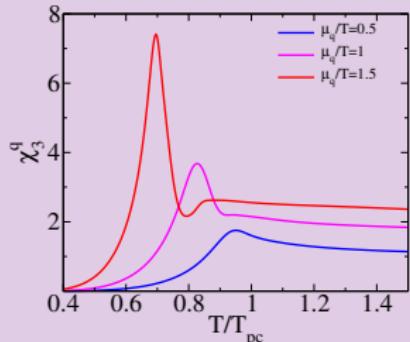
Crossover: $|\partial\sigma/\partial T| > 0.95 \cdot \max(|\partial\sigma/\partial T|)$

NET-QUARK NUMBER DENSITY FLUCTUATIONS $\delta N_q = N_q - \langle N_q \rangle$

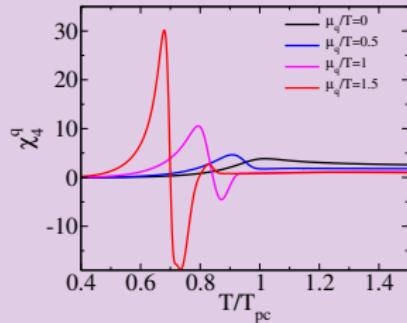
$$\chi_2^q = \frac{1}{VT^3} \langle (\delta N_q)^2 \rangle$$



$$\chi_3^q = \frac{1}{VT^3} \langle (\delta N_q)^3 \rangle$$



$$\chi_4^q = \frac{1}{VT^3} \left(\langle (\delta N_q)^4 \rangle - 3 \langle (\delta N_q)^2 \rangle^2 \right)$$



V.S., B. Friman and K. Redlich '11

- χ_2^q : non-monotonic structure (diverges at CEP)
- χ_4^q : **negative** for nonzero μ_q

KURTOSIS OF NET-QUARK NUMBER DENSITY

$$\text{Kurtosis } R_{4,2}^q = \frac{\chi_4^q}{\chi_2^q} = \frac{\langle (\delta N_q)^4 \rangle}{\langle (\delta N_q)^2 \rangle} - 3 \langle (\delta N_q)^2 \rangle$$

(S. Ejiri, F. Karsch and K. Redlich '05):

quark content of effective degrees of freedom that carry baryon number

- **Low temperature phase:** dominance of effective three-quark states:

$$P_{\text{baryons}}/T^4 \approx \sum_i F(m_i/T) \cosh(3\mu_q/T)$$

$$\leadsto R_{4,2}^q = 9$$

- **High-temperature phase:**

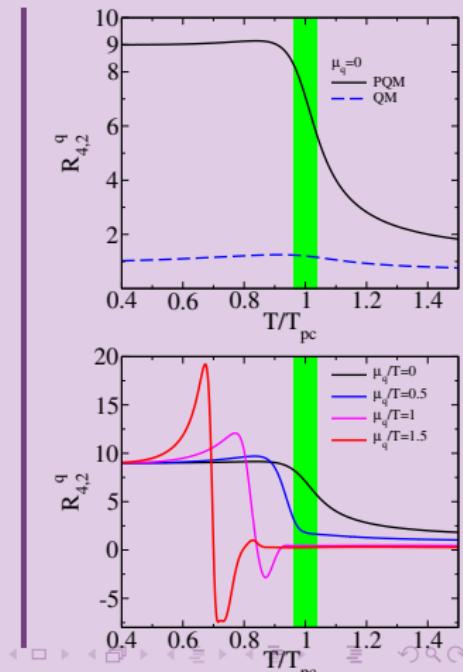
$$P_{q\bar{q}}/T^4 \approx N_f N_c \left[\frac{1}{12\pi^2} \left(\frac{\mu_q}{T} \right)^4 + \frac{1}{6} \left(\frac{\mu_q}{T} \right)^2 + \frac{7\pi^2}{180} \right]$$

$$\leadsto R_{4,2}^q = (6/\pi^2) \approx 1$$

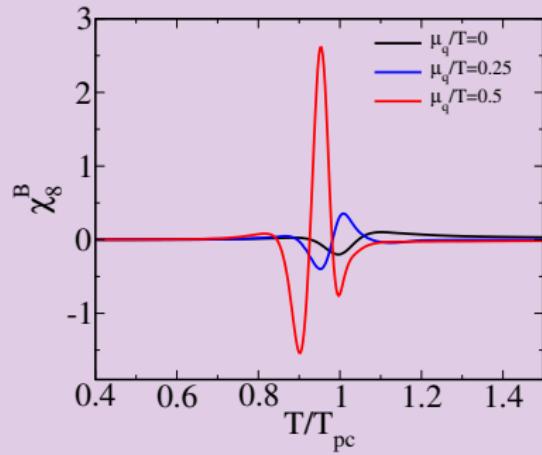
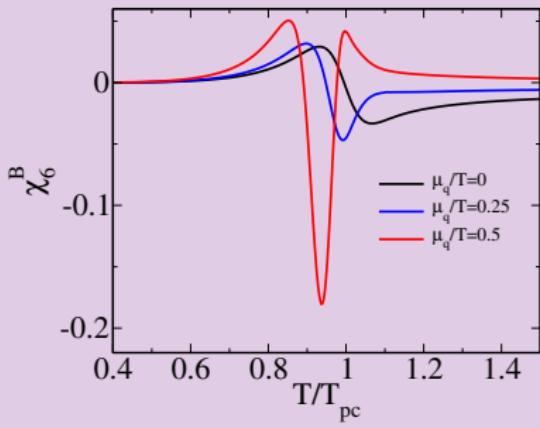
- *PQM: statistical confinement*

- $m_\pi = 0, \mu_q \neq 0$: kurtosis **diverges**

$$R_{4,2}^q \sim \left(\frac{\mu_q}{T} \right)^4 / t^{2+\alpha} \quad (t \propto \text{distance to chiral critical line})$$



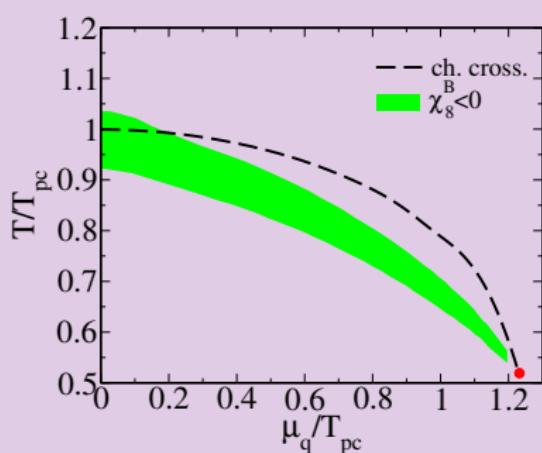
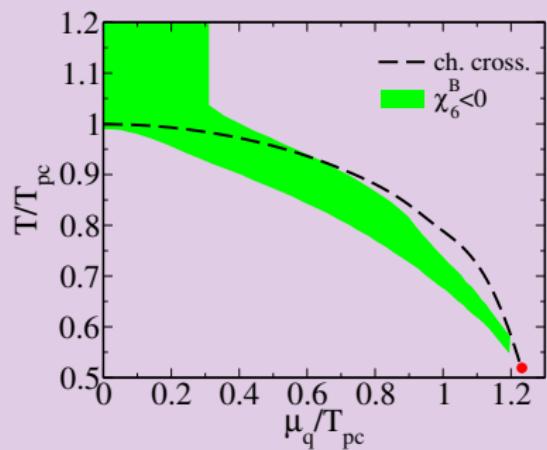
HIGH-ORDER CUMULANTS OF THE BARYON NUMBER DENSITY



- Negative also at $\mu_q = 0$
- Many other constraints from O(4) scaling: B. Friman et. al. '11

HIGHER ORDER BARYON CUMULANT

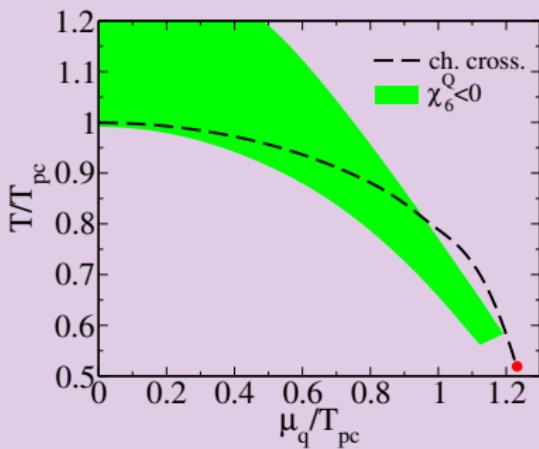
Temperature interval of negative cumulants closest to hadronic phase:



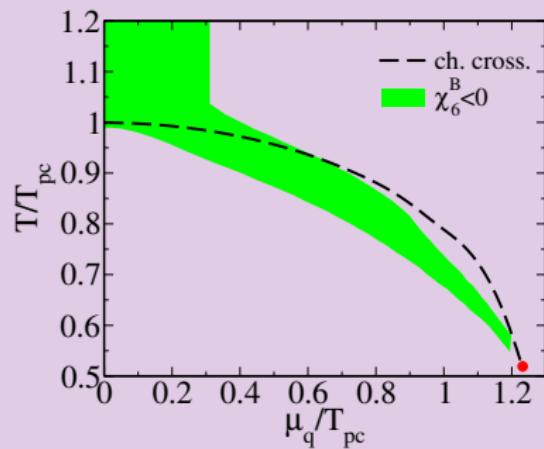
- Negative values (in broken phase!) of high-order cumulants: indicates proximity of freeze-out to crossover

ELECTRIC CHARGE FLUCTUATIONS

Electric charge:

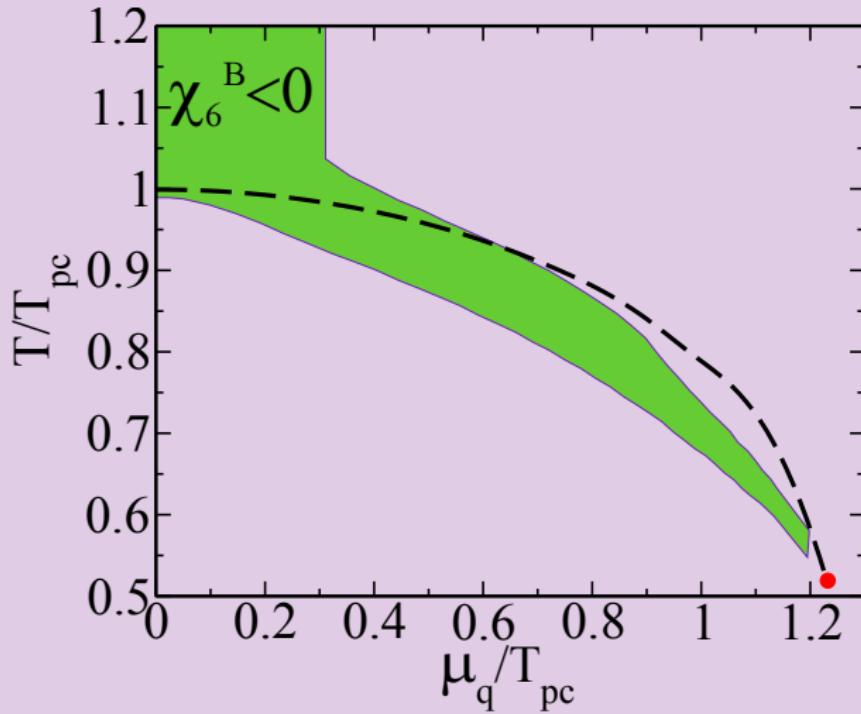


Baryon charge:

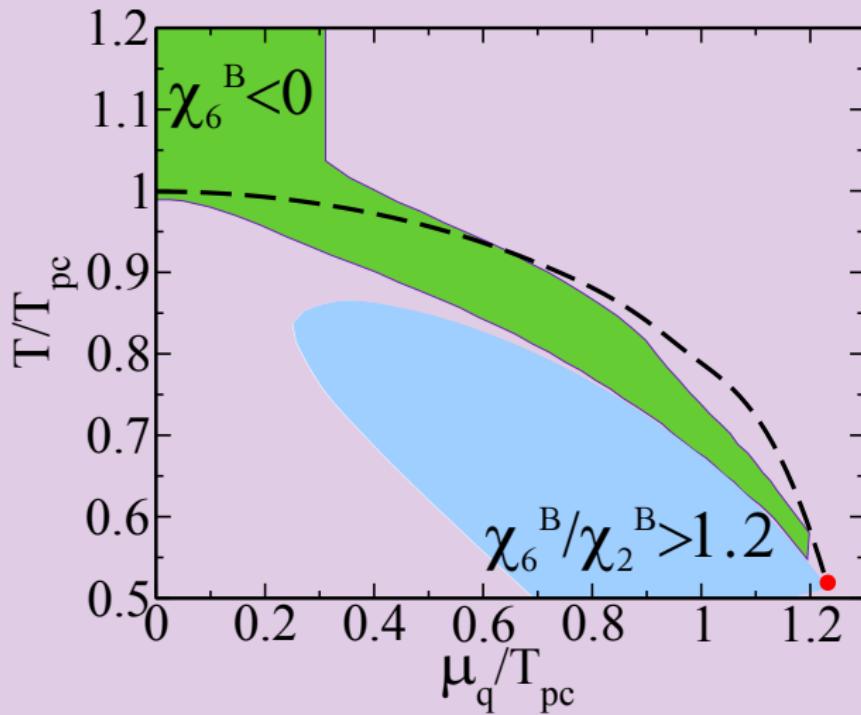


Electric charge fluctuations follow similar pattern as baryon fluctuations

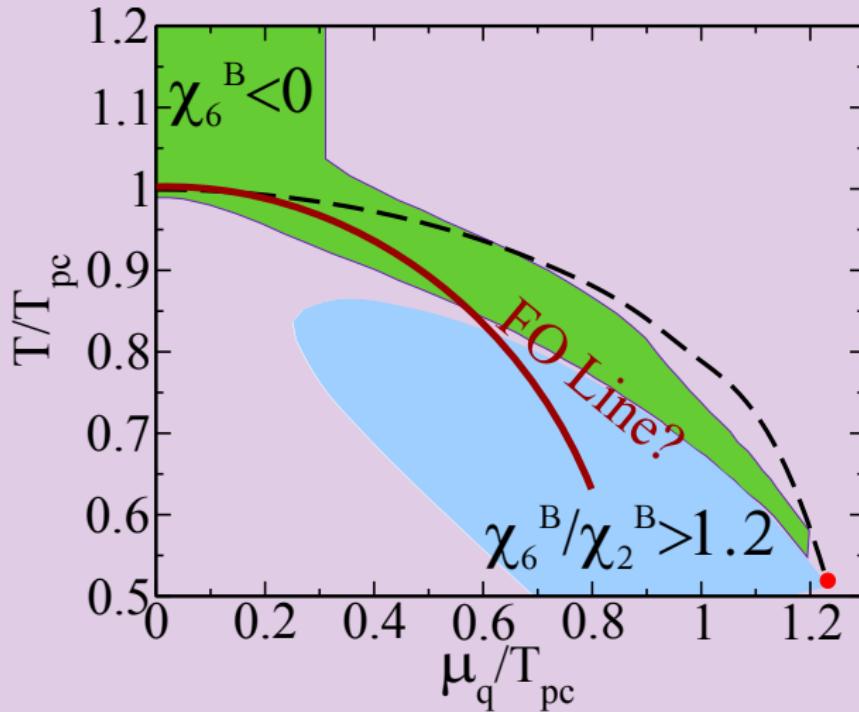
FO SCENARIOS



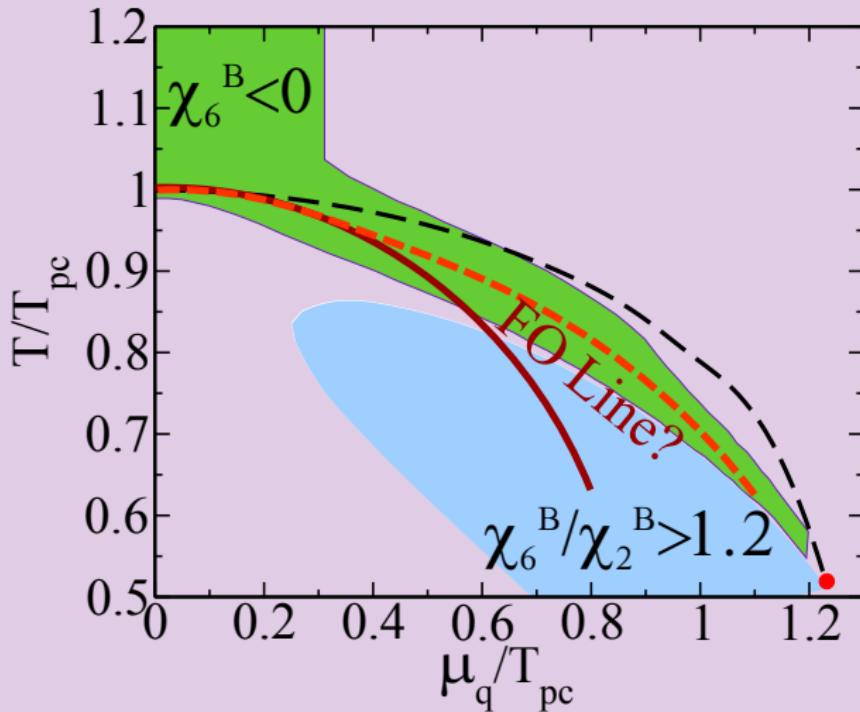
FO SCENARIOS



FO SCENARIOS



FO SCENARIOS



- Models are unable relate FO line and PT line
- Low energies: cumulants might be affected by conservation laws

CONCLUSIONS

- **Fluctuations** of conserved charges are **sensitive** probes of the phase structure. They carry information about deconfinement and chiral phase transitions. Fluctuations can be used to identify order and universality class of phase transition.
- **Negative** χ_6^B and χ_8^B can serve as indication of proximity of chemical freeze-out to crossover line.
- Chiral crossover \leadsto qualitative differences from Hadron Resonance Gas

Precision in $P(N)$ needed to detect negative χ_6^B ?

- pressure $\sim P(N)$
- difference between $P(N)$ with/without chiral crossover
- \sim estimate for number of events

Thank you for attention

Collaborators: B. Friman, F. Karsch and K. Redlich

Backup slides

Functional Renormalization Group

- $p(T, \mu, \textcolor{red}{k})$, $\textcolor{red}{k}$ defines IR cut off \sim
 $p(T, \mu, \textcolor{red}{k})$ includes modes with momentum $> \textcolor{red}{k}$.
- Functional renormalization group equation (exact and general):

$$p(T, \mu, \textcolor{red}{k} - dk) = p(T, \mu, \textcolor{red}{k}) + \boxed{\text{Exact FRG flow}}$$

- Iterating towards $\textcolor{red}{k} \rightarrow 0$: $p(T, \mu, k = 0)$ includes all momentum modes
- Exact FRG is useless, approximations (leading order in gradient expansion):

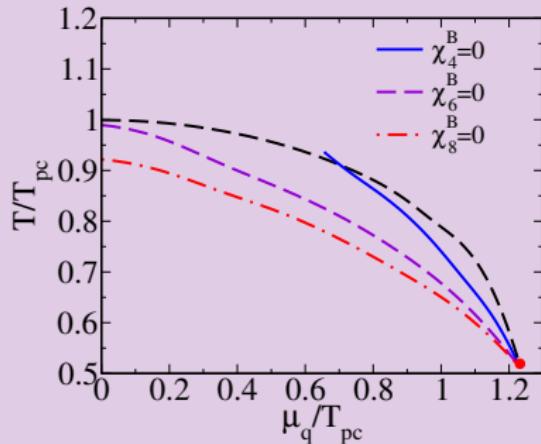
$$p(T, \mu, \textcolor{red}{k} - dk) = p(T, \mu, \textcolor{red}{k}) + \boxed{\text{Approximate FRG flow}}$$

FRG review: J. Berges, N. Tetradis & C. Wetterich, Phys.Rept.363:223-386, '02

FRG formulation of PQM model: V. S., B. Stokic, B. Friman & K. Redlich, PRC, '10

ZEROS OF BARYON CUMULANTS

First zero (closest to hadronic phase) of χ_n^B :



- HRG: $\chi_n^B > 0$
- PQM: $\chi_4^B > 0$ for small μ_q/T
- Zeros of χ_4^B close to CEP: 3d Ising model scaling constraints on singular part (M. Stephanov '11)
- Zeros of high-order cumulants at $\mu_q \approx 0$: close to crossover temperature

FUNCTIONAL RENORMALIZATION GROUP

The general flow equation for the effective action

$$\partial_k \Gamma_k[\Phi, \psi] = \frac{1}{2} \text{Tr} \left\{ \partial_k R_{kB} \left(\Gamma_k^{(2,0)}[\Phi, \psi] + R_{kB} \right)^{-1} \right\} - \text{Tr} \left\{ \partial_k R_{kF} \left(\Gamma_k^{(0,2)}[\Phi, \psi] + R_{kF} \right)^{-1} \right\}$$

The flow equation for the PQM model

$$\begin{aligned} \partial_k \Omega(k, \rho \equiv \frac{1}{2}[\sigma^2 + \pi^2]) &= \frac{k^4}{12\pi^2} \left\{ \frac{3}{E_\pi} \left[1 + 2n_B(E_\pi; T) \right] + \right. \\ &\quad \left. \frac{1}{E_\sigma} \left[1 + 2n_B(E_\sigma; T) \right] - \frac{4N_f N_c}{E_q} \left[1 - N(\ell, \ell^*; T, \mu_q) - \bar{N}(\ell, \ell^*; T, \mu_q) \right] \right\} \end{aligned}$$

$n_B(E; T)$ is the boson distribution functions

$N(\ell, \ell^*; T, \mu_q)$ are fermion distribution function modified owing to coupling to gluons

E_σ and E_π are the functions of k , $\partial\Omega/\partial\rho$ and $\rho\partial^2\Omega/\partial\rho^2$

$$E_q = \sqrt{k^2 + 2g\rho}$$

FRG defines $\Omega(k, \rho; T, \mu_Q, \mu_B)$.

Physically relevant quantity is the thermodynamical potential

$\bar{\Omega}(T, \mu_Q, \mu_B) \equiv \Omega(k \rightarrow 0, \rho \rightarrow \rho_0; T, \mu_Q, \mu_B)$, where ρ_0 is the minimum of Ω .

HEAVY-ION COLLISIONS

- thermodynamical limit
- assumption of full equilibrium
- survival of fluctuations after chemical freeze-out (mainly affects p_T -fluctuations)
- at low collisional energies:
baryon and electric charge fluctuations constrained by conservation laws

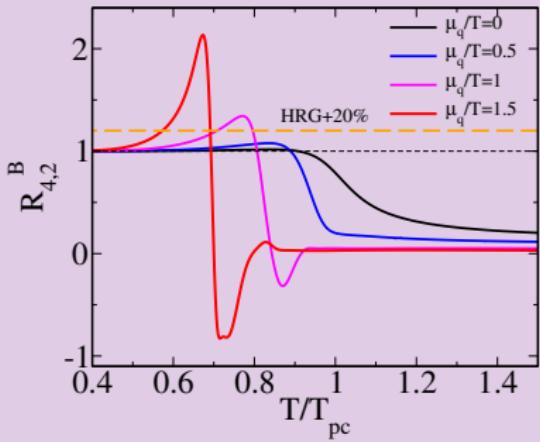
DEVIATION FROM HRG

- HRG:

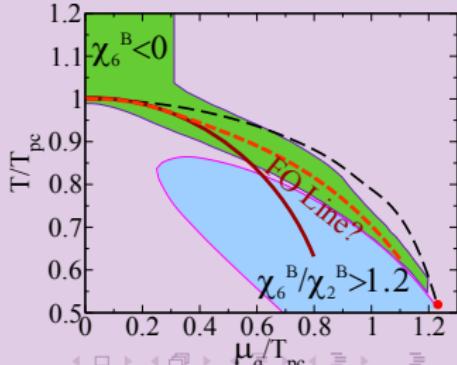
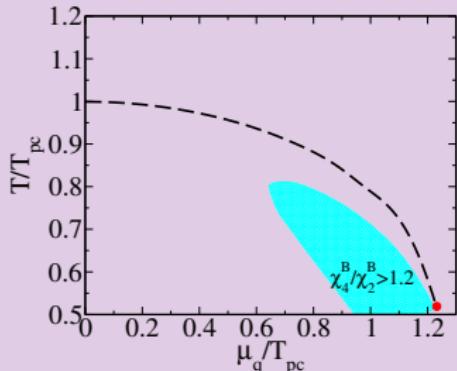
$$R_{4,2}^B \equiv \chi_4^B / \chi_2^B = 1$$

$$R_{6,2}^B \equiv \chi_6^B / \chi_2^B = 1$$

- The FRG PQM model:

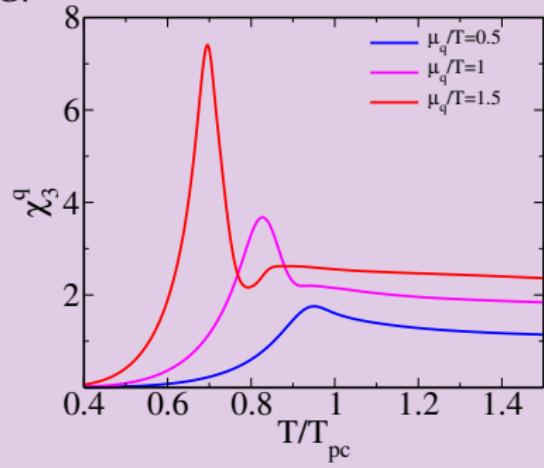


20% increase of cumulant ratios over HRG

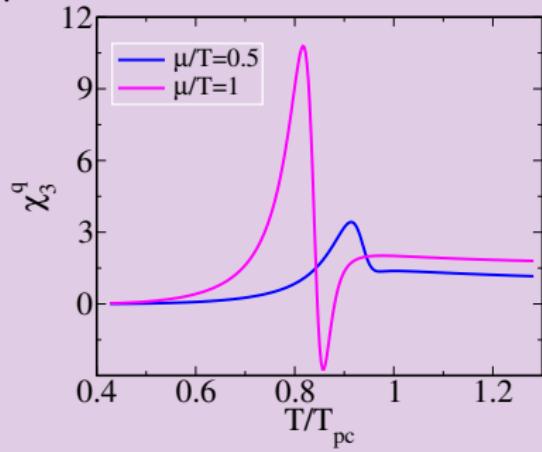


3D ORDER CUMULANT

FRG:



MF:



V.S., B. Friman and K. Redlich 1008.4570 (PRC'11)

FLUCTUATIONS

- Fluctuations of net-quark number χ_n^q and net-baryon charge χ_n^B

$$\chi_n^q = \frac{\partial^n(P/T^4)}{\partial(\mu_q/T)^n} \quad | \quad \chi_n^B = \frac{\partial^n(P/T^4)}{\partial(\mu_B/T)^n} = \left(\frac{1}{3}\right)^n \chi_n^q$$

- Fluctuations of electric charge χ_n^Q

$$\chi_n^Q = \frac{\partial^n(P/T^4)}{\partial(\mu_Q/T)^n}$$

Properties:

- At CEP: for $n \geq 2$, $\chi_n \propto \xi^{n\beta\delta/\nu-3} \approx \xi^{5n/2-3}$, e.g. $\chi_4 \sim \xi^7$
(M. Stephanov '09)
- $m_\pi = 0$ (critical line):
at $\mu_B = 0$, $\chi_n \propto \xi^{(n+2\alpha-4)/(2\nu)}$ for even $n \geq 6$, e.g. $\chi_6 \sim \xi^{1.1}$
at $\mu_B \neq 0$, $\chi_n \propto \xi^{(n+\alpha-2)/\nu}$ for $n \geq 3$, e.g. $\chi_3^B \sim \xi^{1.1}$
- $m_\pi \neq 0$: rapid change in crossover region
(B. Friman, F. Karsch, K. Redlich and V.S. '11)